IMPROVING PMP AND PMF HYDROLOGY USING MONTE CARLO METHODS

C Ryan[1], R Nathan[2]
[1]Catchment Simulation Solutions, NSW
[2]The University of Melbourne, VIC

Introduction

Probable Maximum Precipitation (PMP) estimation in Australia is based on long-established methods: the Generalised Short Duration Method (GSDM) for durations up to 6 hours (published 2003), the Generalised Tropical Storm Method Revised (GTSMR, 2005), and the Generalised Southeast Australia Method (GSAM, 2006).

Book 8 of ARR (2019) has provided a robust update to these prior guidelines. It also collates important earlier work by Nathan & Weinmann (2004), Nathan et al. (2011), and Jordan et al. (2005). These works have described the complexities of the PMF derivation and promoted the use of ensembles in the discipline.

The PMP Flood is the flood resulting from the PMP under probability-neutral conditions. The PMP can be assigned an estimated Annual Exceedance Probability (AEP) based on the catchment area (ARR Book 8, Figure 8.3.2) with the AEP generally increasing (less rare) with increasing catchment area.

The Probable Maximum Flood (PMF) is a hypothetical flood *whose magnitude is such that there is a negligible chance of it being exceeded* (ARR Book 8 6.4.2). In deriving the PMF, *the probability neutral objective for selection of design inputs is explicitly rejected in favour of adopting conservatively high estimates* (ARR Book 8, Section 6.4.2). Pilgrim and Rowbottom (1987) defined the PMF as the limiting value of a flood that could *reasonably* be expected to occur. The reasonableness of the PMF is a key consideration, and concerns around the difficulties of estimating the PMF and ascertaining its reasonableness have long been recognised (ARR Book 8 6.4.1).

For PMF estimates used as an upper limit for potential floodplain inundation, the general guidance is to use loss rates of 0mm and 1mm/hr, the max of a sample of ten temporal patterns and any storages at their full levels (ARR Book 8, Section 6.4.3). However, for design estimates, it is recommended that more consideration be given to the reasonableness of the assumptions.

One method for estimating the reasonableness of the PMF is using Monte Carlo (MC) techniques. Using MC, the reasonableness of the PMF can be considered by calculating what proportion of times the PMF is exceeded when modelling the PMP and stochastically sampling all design inputs from their expected distributions. This exceedance probability may be expected to be between 1-10% (ARR Book 8, Section 6.4.4). This concept is illustrated in Figure 1.

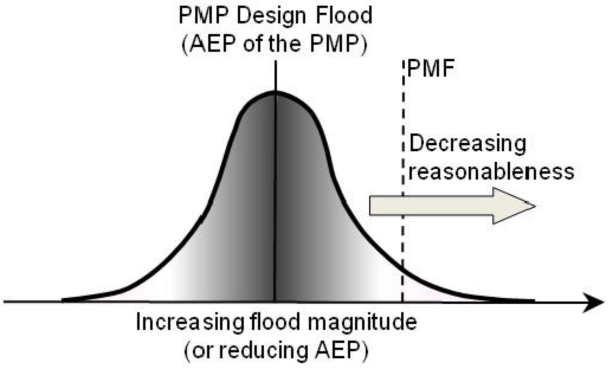

Figure 1: Assessing ‘reasonableness’ based on exceedance (Nathan et al. 2011)

With improvements in computational power and the increasing availability of MC-enabled hydrologic modelling tools, it is worth considering if MC assessment of PMP rainfall should be used in the first instance rather than employed afterwards to test the reasonableness of PMF estimates.

This paper proposes a shift in PMF estimation: rather than selecting conservative model inputs and subsequently testing reasonableness, the PMF may be defined directly as a specified exceedance probability derived from Monte Carlo simulation of PMP rainfall with stochastic sampling of all other inputs. The advantage of this approach is that only a single number has to be selected (rather than a number of PMF input parameter assumptions), and it has a meaningful statistical definition that can be readily understood by stakeholders.

This concept will be demonstrated using a theoretical catchment and storage in a case study.

Tools

MC methods from the Storm Injector software (Catchment Simulation Solutions) are used for the case study. These allow both stratified rainfall sampling with analysis by Total Probability Theorem (TPT) as well as a Static Rainfall Monte Carlo (SRMC) approach with sampling of other parameters and a simple exceedance probability analysis. PMP rainfall depths for subareas were determined using the CatchmentSIM software (Catchment Simulation Solutions).

Case Study

A case study of a theoretical 174 km^2 catchment in the NE of NSW is presented. The location is shown as a red point in Figure 2. A GSDM and GTSMR analysis was completed to determine spatially distributed rainfall depths for short and long durations. For extreme events, including the PMP, critical durations were found to be less than 6 hours, and GSDM rainfalls were applied. GSDM PMP rainfall estimates for 6 hours ranged from 390 to 660 mm for ellipses A-F after application of a 0.74 Moisture Adjustment Factor (MAF).

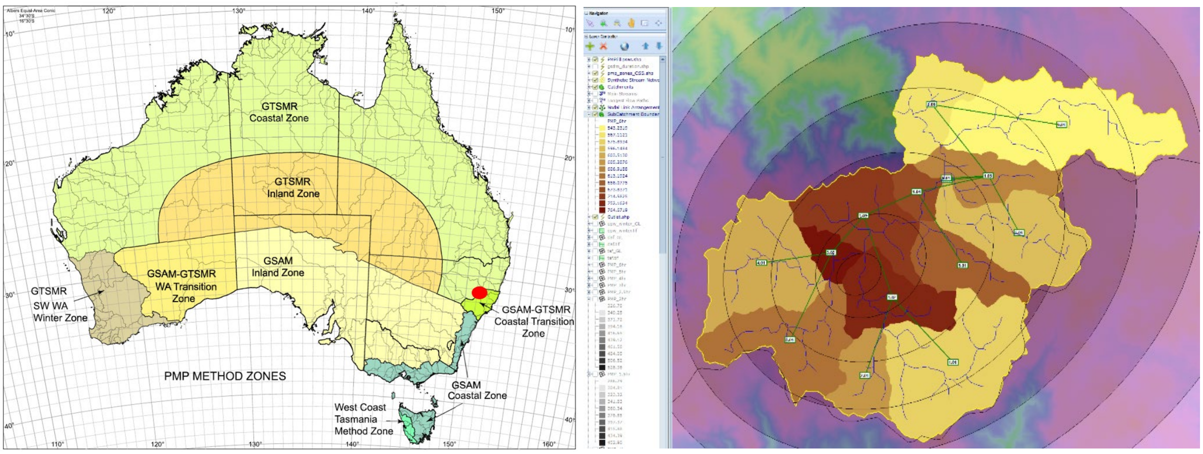

Figure 2: Theoretical Catchment Location and GSDM Rainfall

GSDM PMP rainfall was spatially varied across each subarea, as shown in Figure 2. PMP rainfall totals for each subarea were entered into Storm Injector, where they were used for PMP/PMF modelling and for interpolating rainfall depths for a range of extreme events (1 in 10,000 through 1 in 2.5 million).

An exceedance probability for the PMP was determined to be approximately 1 in 5.7 million (ARR Book 8, Figure 8.3.2) based on a catchment area of 174 km^2.

PMP and PMF hydrology were applied to the case study for scenarios with and without a storage.

Case Study without Storage

Ensemble and stratified MC modelling were undertaken using ARR rare temporal patterns as well as Jordan (2005) extreme temporal patterns for events rarer than 1 in 2000 years. Figure 3 shows the box and whisker plots for these ensembles combined with the stratified MC analysis (samples and polynomial line of fit). The two methods agree well with peak flow increasing rapidly for events rarer than 0.001% AEP.

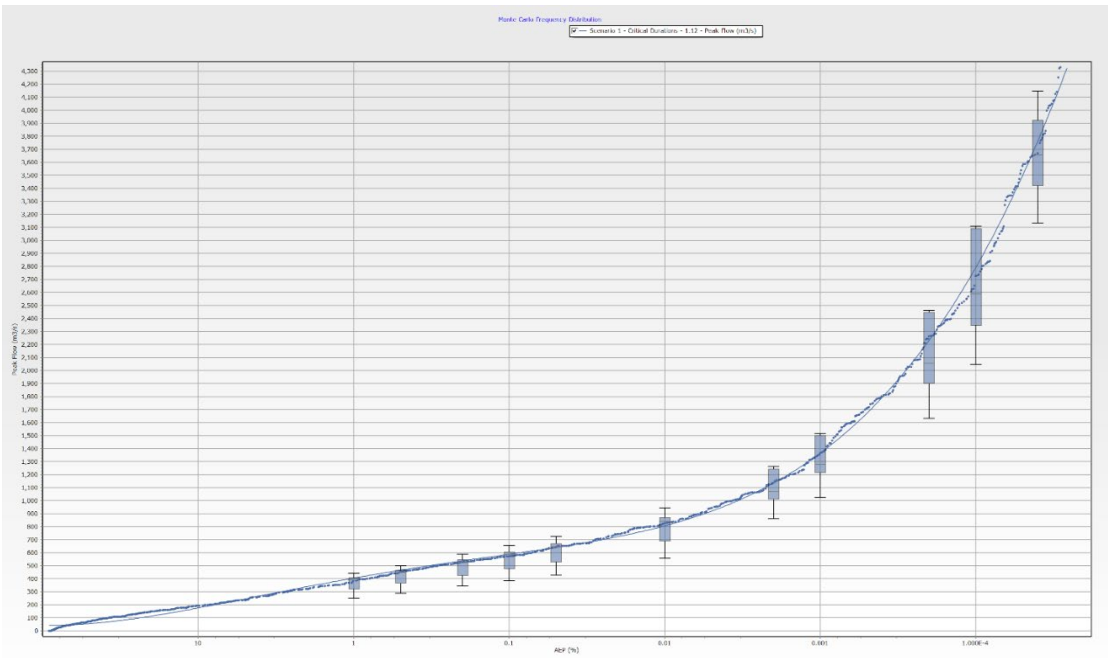

Figure 3: Stratified Monte Carlo Analysis vs Ensembles

The critical duration for events were found to shorten as AEPs became more extreme with 4 – 4.5 hours being critical for events approaching the PMP.

Estimates for the PMP Flood and PMF using the single GSDM pattern were developed and are shown in Figure 4. Ensembles developed using the Jordan (2005) ensembles are also shown in Figure 4. The PMP Flood was estimated using Data Hub losses of 3 mm/hr and Probability Neutral burst losses of approximately 8 mm. The PMF was estimated using 0 mm initial loss and 1 mm/hr continuing loss as suggested by ARR Book 8, Section 6.4.4.

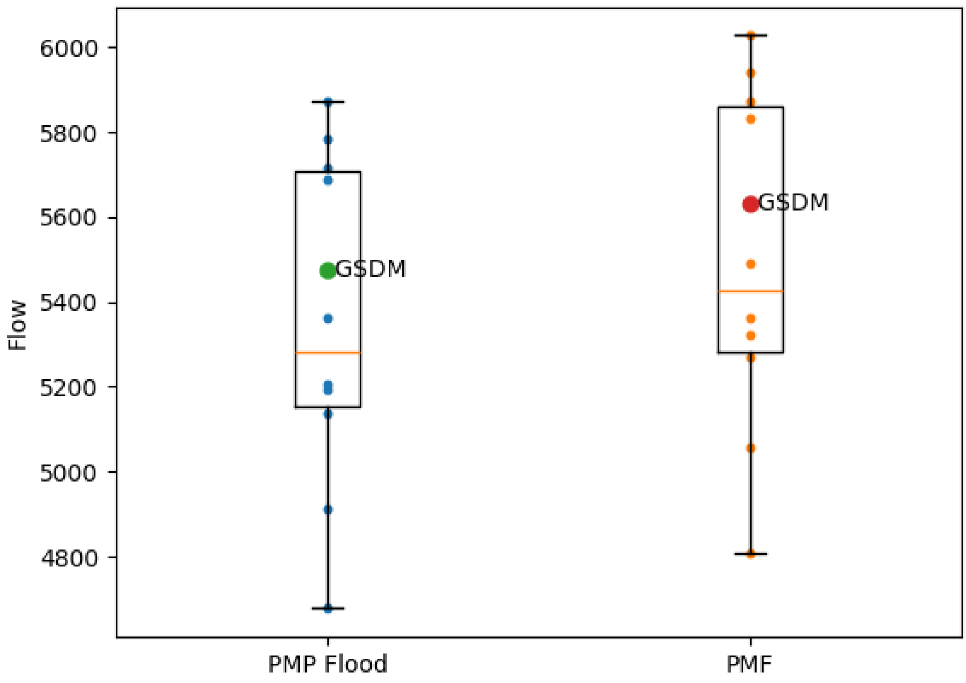

Figure 4: PMP Flood vs PMF ensembles (4.5 hr) vs GSDM single pattern (4 hr)

The differences between the PMP Flood and the PMF ensembles are due to the different loss assumptions which make a relatively small difference (160 m3/s, ~ 3%). These peak flow values seem broadly consistent with worldwide maxima for this catchment size as shown in Figure 5 (refer red point).

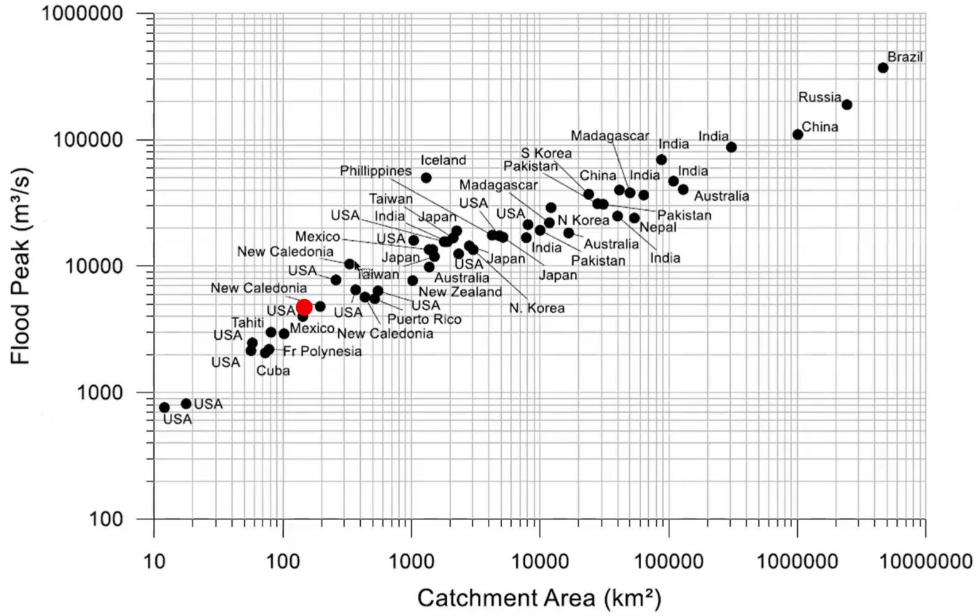

Figure 9-2: World maxima as a function of catchment area (adapted from Hershey, 2001).

Figure 5: PMF Flow versus World Maxima

A SRMC approach was applied using the PMP rainfall estimate. Temporal patterns were stochastically sampled from the Jordan (2005) patterns and loss rates were sampled based on standardised Loss Factors (ARR Book 5, Table 5.3.13) combined with the Data Hub loss rates. Initial and continuing losses were independently stochastically sampled. Figure 6 shows the exceedance probability distribution of the SRMC analysis, the PMP Flood and PMF ensembles, and the single-pattern GSDM result. Ensemble results are horizontally offset for clarity.

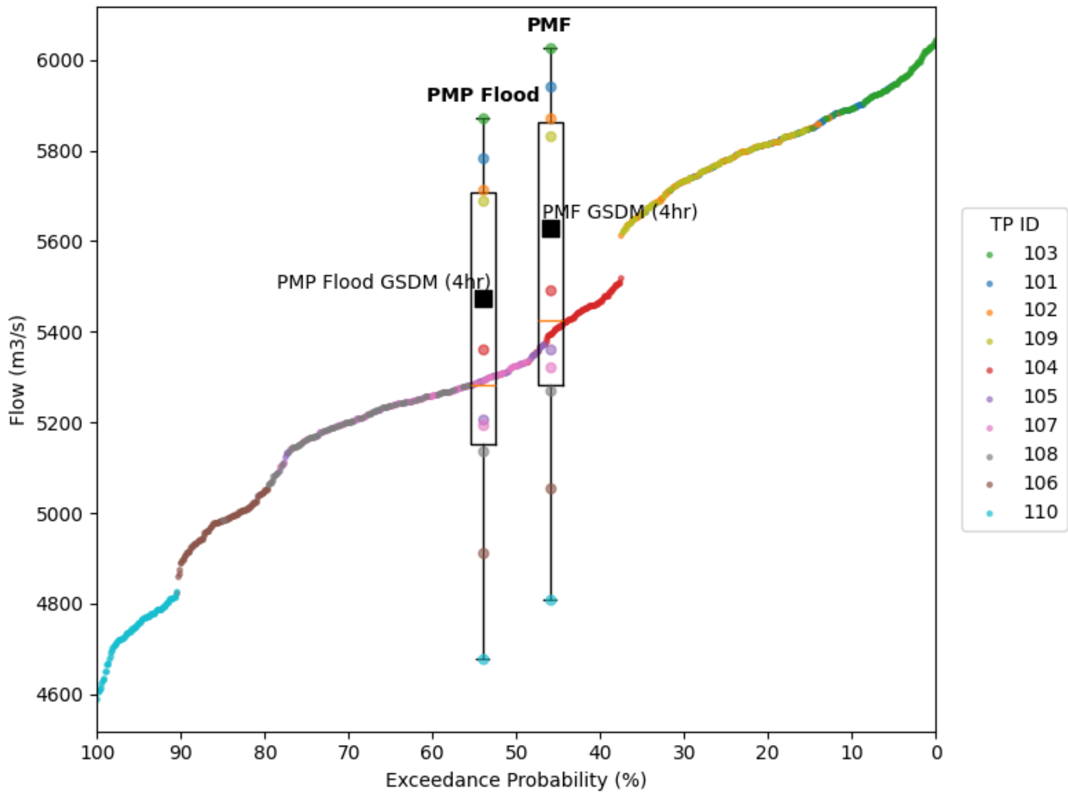

Figure 6: Static Monte Carlo Assessment versus Ensembles

Figure 6 illustrates that the range of values from the SRMC analysis are similar to the ensembles. Ensemble and SRMC results can be seen to be highly dependent on the temporal pattern as shown by the colour-coding. For example, temporal pattern 103 (dark green) produces the highest results in both the ensembles and SRMC results.

Both the PMP/PMF ensembles and SRMC results provide more context on the range of flows that can result from PMP rainfall than the single-pattern GSDM approach. Despite this improvement over the single-pattern GSDM approach, it may be considered for this case study that the SRMC approach does not add a great deal of additional information to the analysis since the ensembles already fully represent temporal pattern variability, and loss rates have been previously shown to have a modest impact on the results. However, for case studies with greater dependence on loss rates or catchments with storages, the SRMC approach may prove more useful. As a demonstration, a theoretical storage was added to the catchment outlet, and the exercise was repeated.

Case Study with Storage

Storage Details

The hydrologic model was augmented with a theoretical 22 GL Storage with a 60m weir and a low-flow outlet at the model outlet. Hydrologic modelling was completed with the storage Initial Water Level (IWL) set at Full Supply Level (FSL) as well as stochastic sampling from a theoretical Cumulative Density Function (CDF) of water levels.

Storage at Full Supply Level

The storage was found to attenuate flows with peak outflow reaching approximately 1,880 m^3/s. Critical durations were longer, particularly for more common events such as the 1% AEP and were 6 hours for PMP/PMF. Ensemble modelling and stratified MC modelling (samples and polynomial line of fit) were applied and found to agree well, as shown in Figure 7.

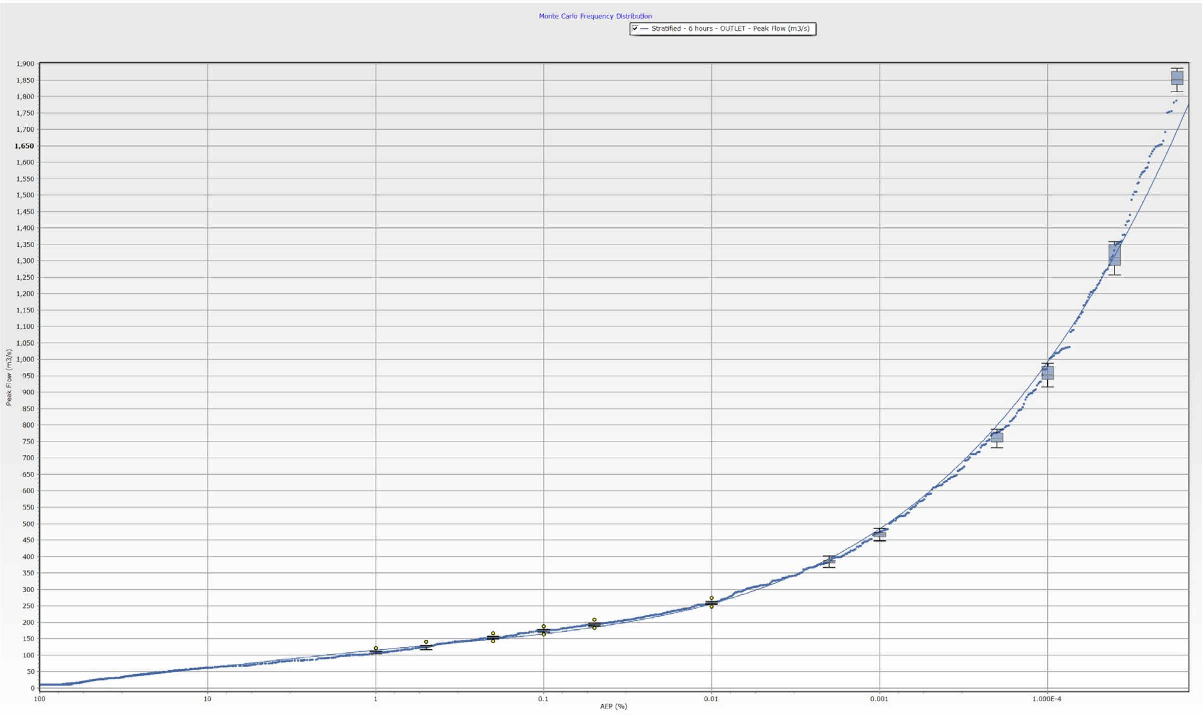

Figure 7: Stratified Monte Carlo Analysis vs Ensembles with Storage at FSL

An SRMC analysis was conducted with the storage at FSL as well as PMP Flood and PMF ensembles, as shown in Figure 8.

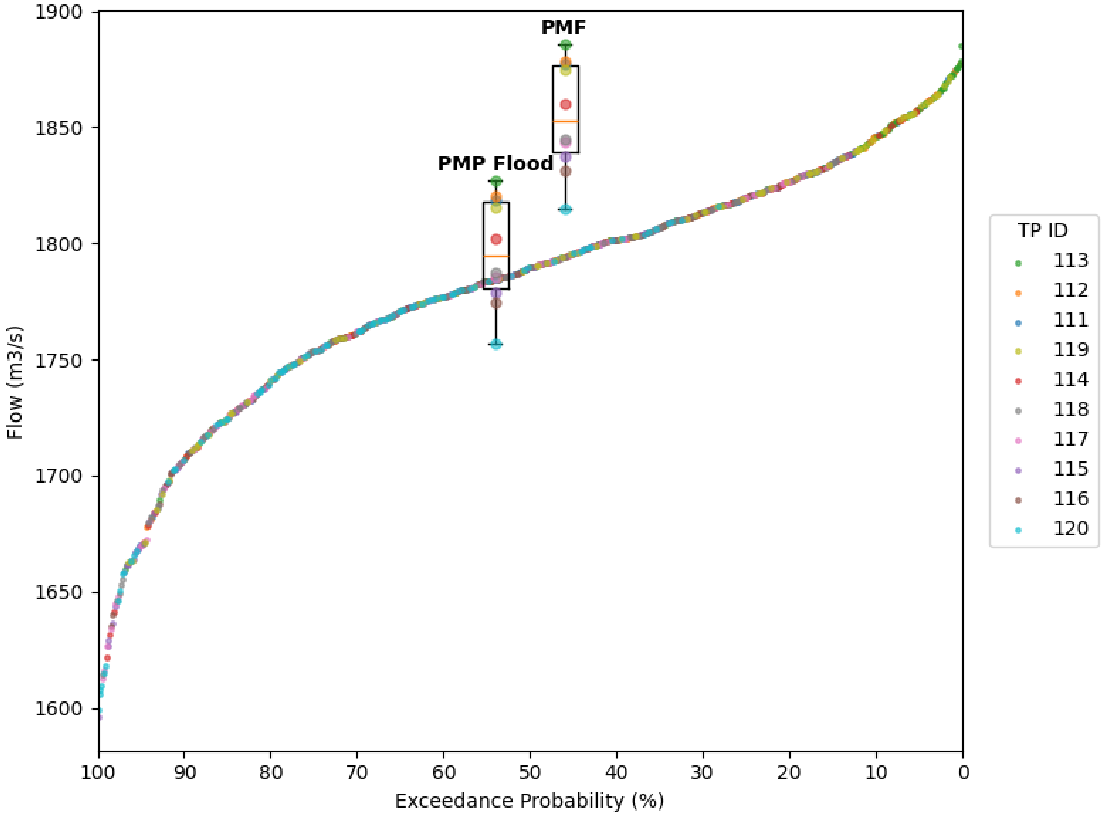

Figure 8: Static Monte Carlo PMP Analysis vs Ensembles (6 hr Full Storage)

Unlike the case study without the storage, the ensemble results do not fully reflect the range of results generated by the SRMC approach. The SRMC results can be outside the range of the PMP Flood ensemble due to the loss rate distribution which is more consequential in the storage scenario where peak flows are attenuated. The PMF ensemble does capture the top of the SRMC results well due to its loss assumptions, which correspond to the extremely low likelihood of the adopted probability distribution.

The SRMC results provide improved context to the ensemble results and allow the reasonableness of the PMF to be directly quantified through its exceedance probability.

This analysis was based on the storage starting at FSL. As a result, these results do not take into consideration the likelihood of this starting condition for the storage. A practitioner may wish to consider the storage starting level based on a probability distribution derived from historical observations or on the likely distribution under operational procedures. MC is useful for this purpose as the ensemble approach cannot readily be used with variations in variables such as storage IWL.

Storage with variable Initial Water Level (IWL)

For this example, a theoretical Cumulative Density Function (CDF) was created for the storage IWL as shown in Figure 9. A histogram showing 10,000 random samples projected through the CDF is also shown.

Percent of Time	Level (m)
0	94
0.1	94.5
0.29	95
0.7	95.5
1.44	96
3.5	96.5
7.23	97
13.22	97.5
21.94	98
29.41	98.25
38.18	98.5
48.09	98.75
50	98.79
58.86	99
72.76	99.25
85.79	99.5
95.85	99.75
100	100

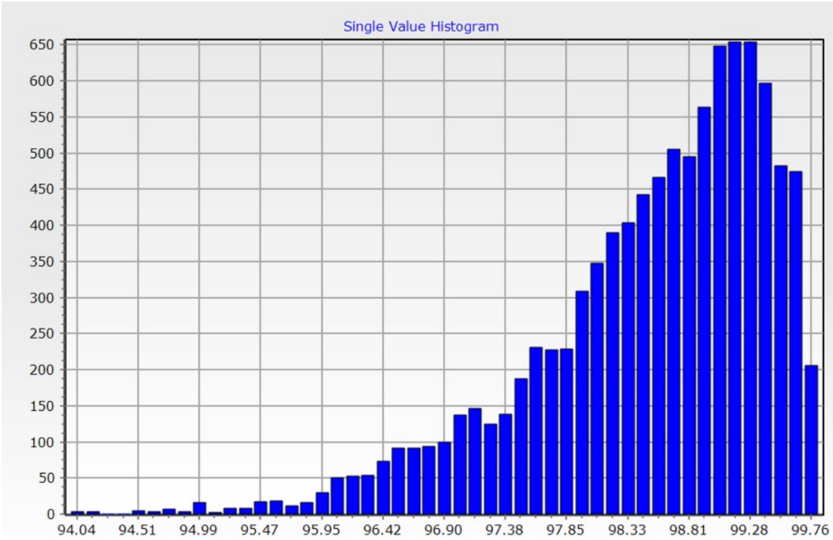

Figure 9: Theoretical CDF for storage IWL

SRMC results for the storage starting at FSL (100m) and stochastic sampling of IWL from the CDF are shown in Figure 10. The 50% exceedance value for the storage IWL is 98.79m and was used for the PMP Flood with variable IWL ensemble.

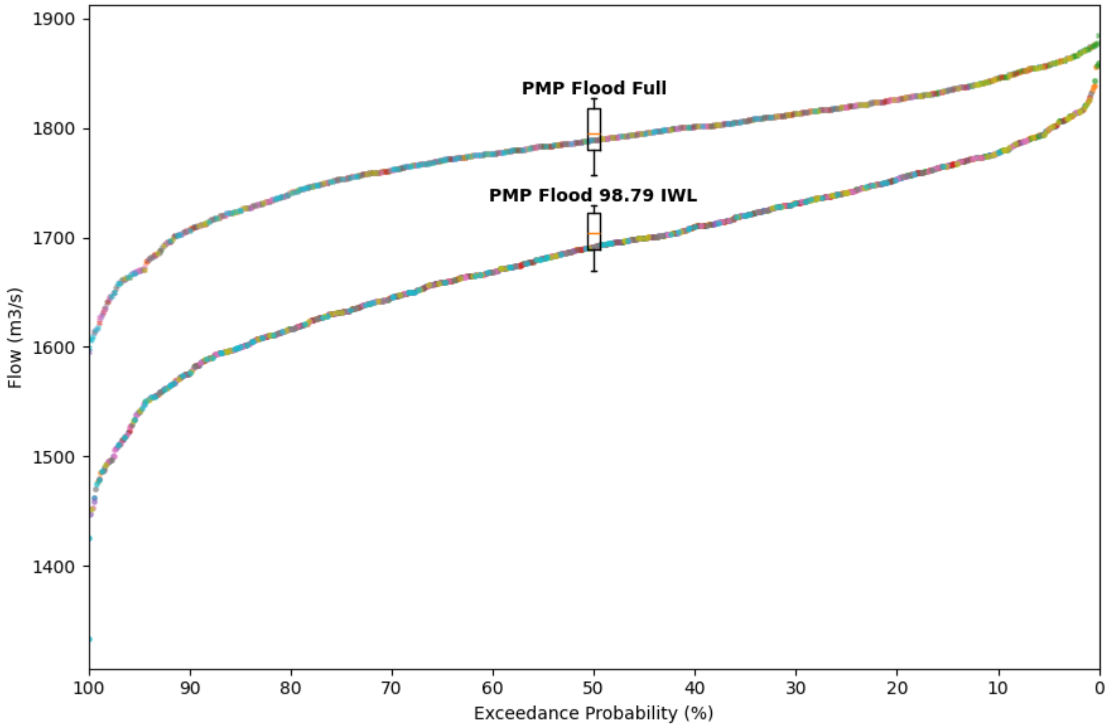

Figure 10: Static PMP MC Analysis for Full vs Variable IWL

Figure 10 shows that the SRMC curve for the variable IWL scenario has a wider range and a steeper slope than in the full storage scenario. The PMP Flood ensemble generated with the 50% exceedance IWL only represents a small part of the curve's range.

As an example of typical PMF assumptions, Nathan et al. (2011) surveyed several different practitioners and their preferred PMF assumptions are listed in Figure 11.

Table 1: Alternative definitions of the deterministic PMF

Option	Pre-burst	Temp Pattern	Initial Loss (mm)	Cont. Loss (mm/hr)	Initial Water Level
A	No	AVM[1]	0	1	FSL
B	Yes	AVM	0	1	FSL
C	No	Max[2]	0	1	FSL
D	Yes	Max[2]	0	1	FSL
E	Yes	Max	0	Med[3]	FSL
F	Yes	10%	90%[4]	90%	10%

1) "AVM" denotes a temporal pattern of average variability
2) "Max" denotes adoption of the temporal pattern that yields the highest (or second highest) flood peak
3) "Med" denotes a continuing loss rate expected to be typically associated with large events
4) "10%" denotes a value that is exceeded 10% of the time, "90%" a value that is exceeded 90% of the time.

Figure 11: Storage scenario PMF assumptions

Three of these scenarios from Figure 11 (Options A, C and F) were modelled and are shown in Figure 12. These scenarios fall at the very top of the SRMC results. This is to be expected, as these assumptions are very conservative and are rarely sampled during typical MC sampling. The most conservative, Option C requires selection of the highest temporal pattern, a storage IWL at FSL and 0,1 loss rates.

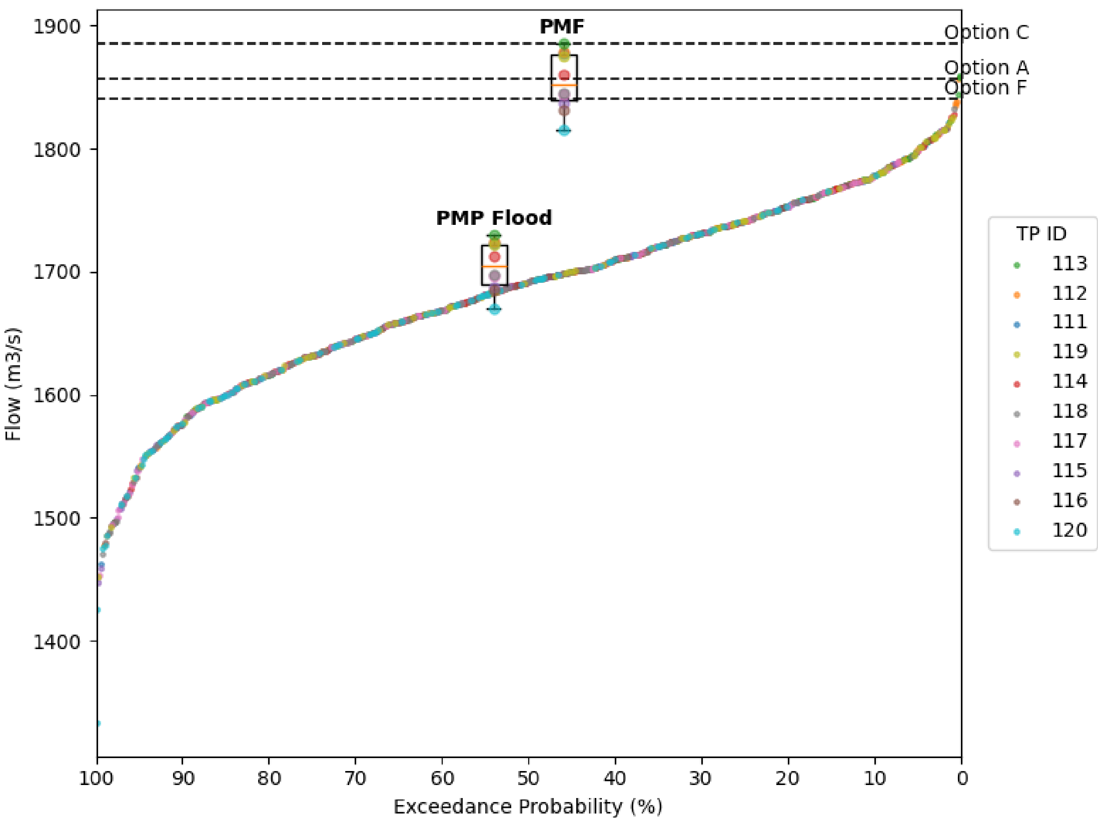

Figure 12: Storage Scenarios vs SRMC (6hr, Variable Water Level)

Figure 13 shows the tail end of the SRMC results, with interpolated peak flows for various exceedance probabilities (10%, 5%, and 1%) as well as the results for the storage scenario options. It can be seen that all 3 of the storage scenarios exceed the 1% exceedance probability of the SRMC results. Option C would have a very small exceedance probability, well below the 1%-10% range that may be considered reasonable with reference to ARR Book 8, Section 6.4.4.

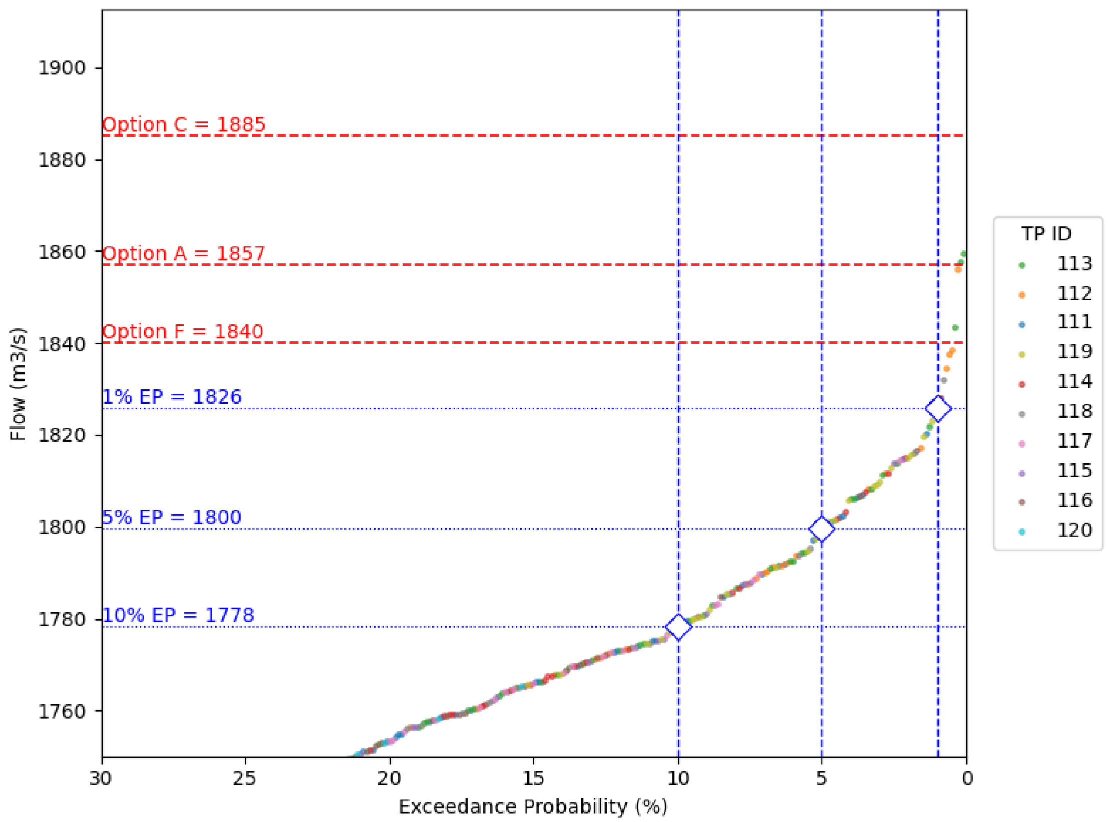

Figure 13: Storage Scenario results vs SRMC Exceedance Probabilities

ARR also asks practitioners to estimate the AEP shift between the PMP Flood and PMF. While the AEP of the PMF is undefined, it is possible to estimate the relative rarity of the PMF compared to the PMP Flood by extrapolating a flood frequency curve out to the flow modelled for the PMF as illustrated in Figure 14.

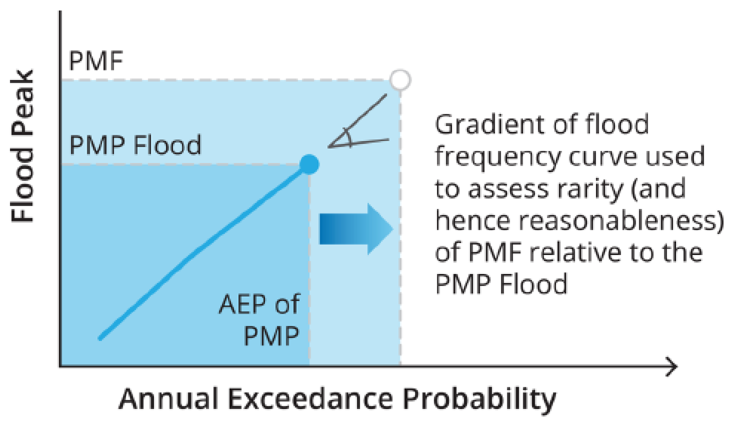

Figure 14: Extending a frequency curve to estimate PMF AEP shift (ARR Figure 8.6.2-b)

In the absence of Flood Frequency Analysis data to inform the extension of a frequency curve, the stratified MC results can be used. Figure 15 below illustrates how a log-linear fit to the extreme end of the stratified MC frequency curve can be used to extrapolate and inform the AEP shift for the Option C storage scenario result of 1,885 m^3/s as compared to the PMP Flood ensemble average. In this case, the PMP Flood AEP was set based on the 1 in 5.7 million from ARR Book 8, Figure 8.3.2, and the PMF extrapolation shifted this AEP to approximately 1 in 9 million.

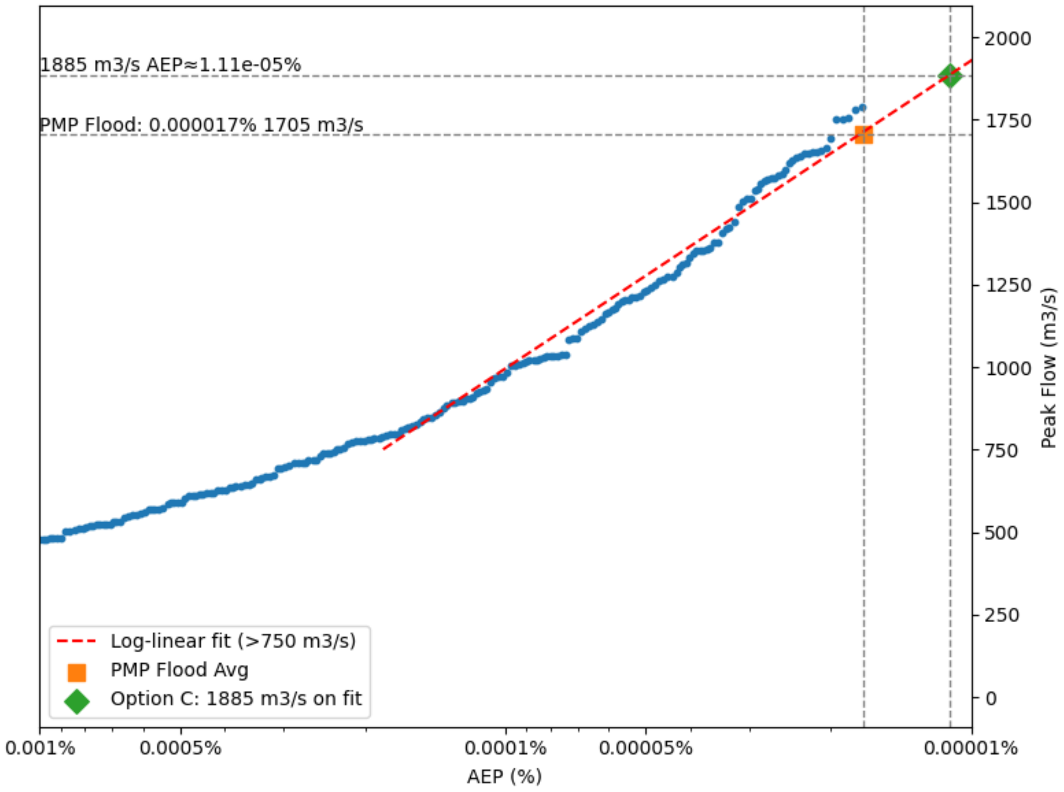

Figure 15: Estimating AEP Shift PMP Flood to PMF (Option C)

Conclusion

This paper advocates the use of Monte Carlo methods using a static PMP rainfall with sampling of other parameters from their probability distributions and estimation of the PMF based on a specific exceedance probability. The selection of a single parameter (the PMF exceedance probability) is seen as preferred to a series of PMF input parameter assumptions which often have an unquantified cumulative effect on the reasonableness of the PMF estimate. Furthermore, this parameter can be related to the use case for the PMF estimate and has a meaningful statistical definition that can be readily understood by stakeholders.

Even in cases where the application of ensembles is seen as a suitable exploration of the range of possible PMF estimates, the SRMC approach can add context and confidence that these ensembles indeed do adequately reflect the range of outcomes and that the ultimate PMF estimate is reasonable.

It is recommended that practitioners consider adopting a Monte Carlo-based exceedance definition of the PMF, particularly for systems with storages or significant parameter uncertainty. This provides a direct link between PMP/PMF modelling decisions and risk, consistent with the broader philosophy of ARR.

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